

8.3

Surface Area and Volume of Prisms and Pyramids



surface area

- the number of square units needed to cover the surface of a three-dimensional object

volume

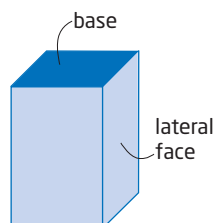
- the amount of space that an object occupies, measured in cubic units

pyramid

- a polyhedron whose base is a polygon and whose faces are triangles that meet at a common vertex

lateral faces

- the faces of a prism or pyramid that are not bases



To package their products economically, manufacturers need to know the amount of material required for the package. To do this, they need to know the **surface area** of the package.

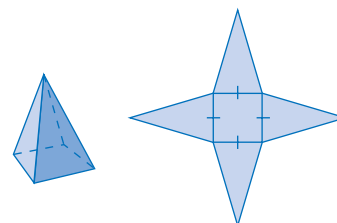
The amount of space a package occupies is the **volume**. Products such as cereal, laundry detergent, and dog food are often sold by mass rather than volume. Why do you think these products are sold by mass?

Investigate

How can you model the surface area and volume of a pyramid?

A: Surface Area

- A square-based **pyramid** and its net are shown.
 - What is the shape of the base? Write a formula for its area.
 - What is the shape of each **lateral face**? Write a formula for the area of one lateral face.
 - Write an expression for the surface area of the pyramid. Simplify the expression to give a formula for the surface area of a pyramid.
- Reflect** How would the results in step 1 change if you were developing a formula for the surface area of a hexagon-based pyramid? an octagon-based pyramid? Describe how to find the surface area of any pyramid.



B: Volume

- Cut the top off the milk carton to form a prism.
 - On a piece of construction paper, draw a net for a pyramid that has the same base and height as the prism.
 - Cut out the net and tape it together to form a pyramid.
- Estimate the ratio of the volume of the prism to the volume of the pyramid.
- Cut along three sides of the base of the pyramid. Fill the pyramid with sand, rice, or another suitable material.
 - Pour from the pyramid into the prism. How many pyramids full of material does it take to fill the prism?
 - What fraction of the volume of the prism is the volume of the pyramid?
- Reflect** What conclusion can you draw about the relationship between the volume of a pyramid and the volume of a prism with the same base and height?



- an empty 250-mL milk carton
- construction paper
- scissors
- tape
- sand, rice, or another suitable material

Optional

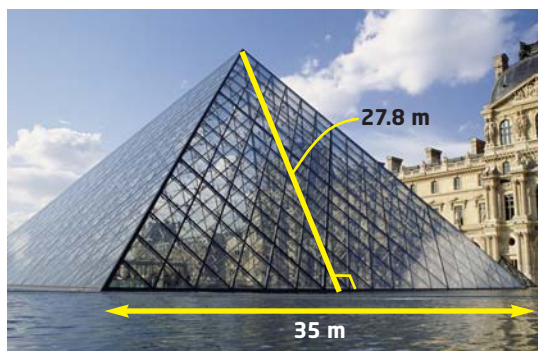
BLM 8.3.1 Net for a Pyramid

BLM 8.3.2 Net for a Prism

Example 1 Surface Area of a Pyramid

A modern example of a pyramid can be found at the Louvre in Paris, France. The glass square-based pyramid was built as an entrance to this famous museum.

Calculate the surface area of the pyramid, including the base area.



Solution

The surface area consists of the square base and the four congruent triangular faces.

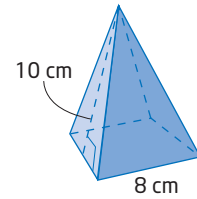
$$\begin{aligned} SA_{\text{pyramid}} &= A_{\text{base}} + 4A_{\text{triangle}} \\ &= (35)(35) + 4 \left[\frac{1}{2} (35)(27.8) \right] \\ &= 1225 + 1946 \\ &= 3171 \end{aligned}$$

Each triangle has a base of 35 m and a height of 27.8 m.

The surface area of the pyramid is 3171 m².

Example 2 Volume of a Pyramid

- a) Determine the volume of the pyramid-shaped container, to the nearest cubic centimetre.
- b) Express the capacity, in litres.

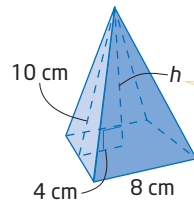


Solution

- a) The volume of any pyramid can be determined using the formula

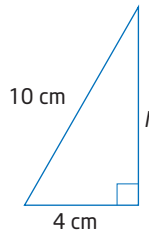
$$V = \frac{1}{3}(\text{area of the base})(\text{height}).$$

First, determine the height of the pyramid using the Pythagorean theorem.



The height of a pyramid is the perpendicular distance from the vertex to the base.

$$\begin{aligned}h^2 + 4^2 &= 10^2 \\h^2 + 16 &= 100 \\h^2 &= 84 \\h &= \sqrt{84} \\h &\doteq 9.2\end{aligned}$$



The slant height of 10 cm is the hypotenuse of the triangle. Since this is the longest side of the triangle, this means that the height of the pyramid is less than 10 cm.

Now, calculate the volume.

$$\begin{aligned}V &= \frac{1}{3}(\text{area of the base})(\text{height}) \\&= \frac{1}{3}(8^2)(9.2) \\&\doteq 196\end{aligned}$$

The volume of the container is about 196 cm^3 .

- b) The capacity is the maximum volume that a container will hold. When a product is packaged, the container is usually not filled to capacity. This may be a factor to consider in some problems.

$$1 \text{ cm}^3 = 1 \text{ mL}$$

The capacity of the container is 196 mL or 0.196 L of liquid.

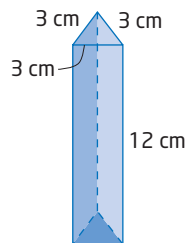
Literacy Connections

For a pyramid, the height of a lateral face is called the *slant height*. The slant height of the pyramid in Example 2 is 10 cm.

Example 3 Surface Area and Volume of a Triangular Prism

Chocolate is sometimes packaged in a box that is shaped like a triangular prism.

- Calculate the amount of material required to make this box, to the nearest square centimetre.
- Calculate the volume of this box, to the nearest cubic centimetre.



Solution

- The amount of material required is the surface area of the prism. The surface area consists of the top and bottom of the box, which are triangles, and the three faces, which are congruent rectangles.

First, determine the height, h , of each triangle.

Use the Pythagorean theorem.

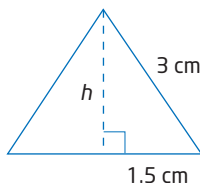
$$h^2 + 1.5^2 = 3^2$$

$$h^2 + 2.25 = 9$$

$$h^2 = 6.75$$

$$h = \sqrt{6.75}$$

$$h \doteq 2.6$$



Now, calculate the surface area.

$$SA = 2A_{\text{base}} + 3A_{\text{face}}$$

$$= 2 \left[\frac{1}{2} (3)(2.6) \right] + 3(3)(12)$$

$$= 7.8 + 108$$

$$= 115.8$$

There are two bases and three lateral faces.

Approximately 116 cm² of material is needed to make the triangular prism box.

- The volume of any right prism can be found using the formula $V = (\text{area of the base})(\text{height})$.

$$\begin{aligned} V_{\text{triangular prism}} &= (\text{area of the base})(\text{height}) \\ &= (3.9)(12) \\ &= 46.8 \end{aligned}$$

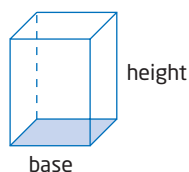
The volume is about 47 cm³.

Literacy Connections

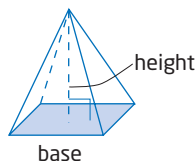
A right prism has two parallel and congruent bases and lateral faces that are perpendicular to its bases. The triangular prism in Example 3 is a right triangular prism.

Key Concepts

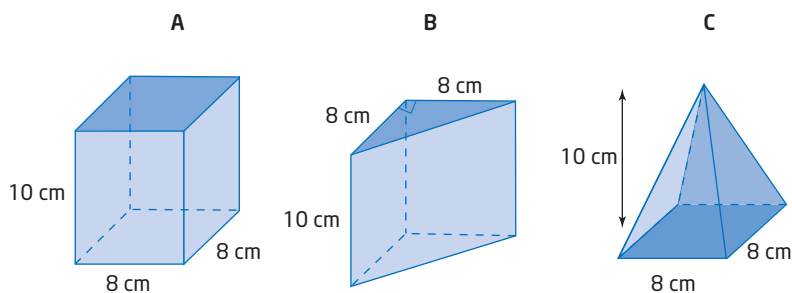
- Surface area is a measure of how much material is required to cover or construct a three-dimensional object. Surface area is expressed in square units.
- The surface area of a prism or pyramid is the sum of the areas of the faces.
- Volume is a measure of how much space a three-dimensional object occupies. Capacity is the maximum volume a container can hold. Volume and capacity are measured in cubic units.
- The litre (L) is a measure of capacity or volume often used for liquids.
 $1 \text{ L} = 1000 \text{ cm}^3$ or $1 \text{ mL} = 1 \text{ cm}^3$.
- For a prism, $\text{Volume} = (\text{area of base})(\text{height})$.



- For a pyramid, $\text{Volume} = \frac{1}{3}(\text{area of base})(\text{height})$.



Communicate Your Understanding

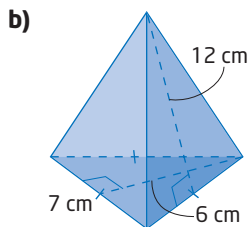
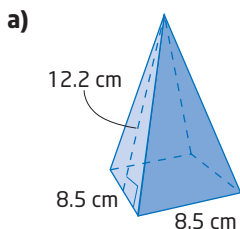


- Describe how the shapes are alike. How are they different?
- Describe how you would determine the volume of each shape.
- Without doing the calculations, predict which shape has the greatest volume. Explain.
- Describe how you would determine the surface area of each shape.
- What unknown values would you need to find to complete the surface area calculations? Explain how you can determine the unknown values.

Practise

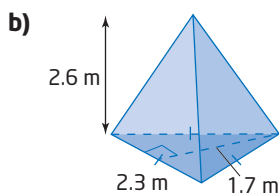
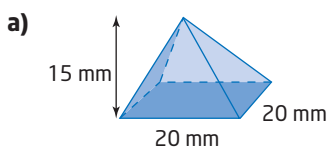
For help with question 1, see Example 1.

1. Determine the surface area of each object.



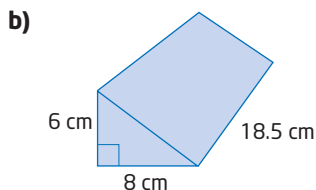
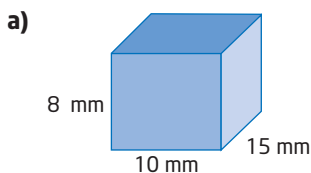
For help with question 2, see Example 2.

2. Determine the volume of each object. Round to the nearest cubic unit, when necessary.

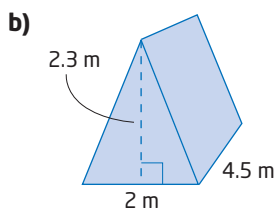
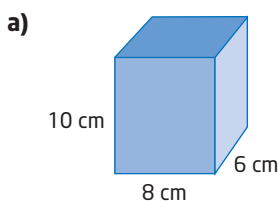


For help with questions 3 to 5, see Example 3.

3. Determine the surface area of each object.



4. Determine the volume of each object.



5. A rectangular prism has length 3 m, width 2 m, and height 4 m.

- a) Determine the surface area of the prism.
b) Determine the volume of the prism.

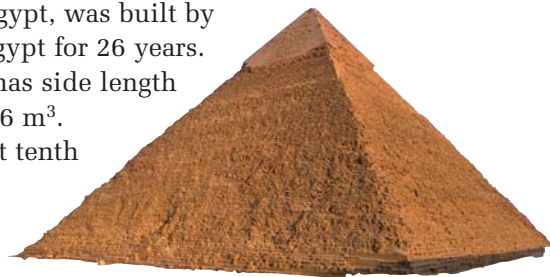
Connect and Apply

6. A cereal box has a volume of 3000 cm^3 . If its length is 20 cm and its width is 5 cm, what is its height?

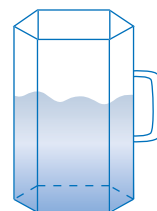


7. Sneferu's North Pyramid at Dahshur, Egypt, is shown. Its square base has side length 220 m and its height is 105 m.
- Determine the volume of this famous pyramid.
 - Determine its surface area, to the nearest square metre.

8. The Pyramid of Khafre at Giza, Egypt, was built by the Pharaoh Khafre, who ruled Egypt for 26 years. The square base of this pyramid has side length 215 m and its volume is $2\,211\,096\text{ m}^3$. Calculate its height, to the nearest tenth of a metre.



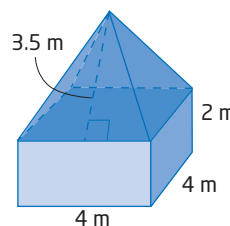
9. The milk pitcher shown is a right prism. The base has an area of 40 cm^2 and the height of the pitcher is 26 cm. Will the pitcher hold 1 L of milk?



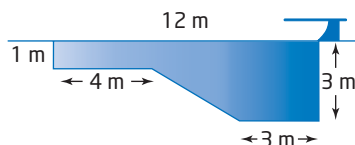
10. A juice container is a right prism with a base area of 100 cm^2 .
- If the container can hold 3 L of juice, what is its height?
 - Describe any assumptions you have made.

11. Adam has built a garden shed in the shape shown.

- Calculate the volume of the shed, to the nearest cubic metre.
- Adam plans to paint the outside of the shed, including the roof but not the floor. One can of paint covers 4 m^2 . How many cans of paint will Adam need?
- If one can of paint costs \$16.95, what is the total cost, including 7% GST and 8% PST?



12. **Chapter Problem** The diagram shows the side view of the swimming pool in Emily's customer's yard. The pool is 4 m wide.

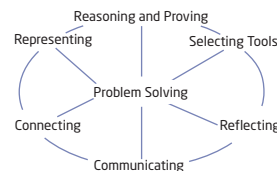


- Estimate how many litres of water the pool can hold.
- Calculate how many litres of water the pool can hold.
- When the pool construction is complete, Emily orders water to fill it up. The water tanker can fill the pool at a rate of 100 L/min. How long will it take to fill the pool at this rate?

13. A triangular prism has a base that is a right triangle with shorter sides that measure 6 cm and 8 cm. The height of the prism is 10 cm.
- Predict how doubling the height affects the volume of the prism.
 - Check your prediction by calculating the volume of the original prism and the volume of the new prism.
 - Was your prediction accurate?
 - Is this true in general? If so, summarize the result.

Achievement Check

14. a) Design two different containers that hold 8000 cm^3 of rice. One should be a rectangular prism and one should be a cylinder.
- b) Determine the surface area of each one, to the nearest square centimetre.
- c) Which shape would you recommend to the manufacturer and why?



Extend

15. A pyramid and a prism have congruent square bases. If their volumes are the same, how do their heights compare? Explain.
16. A statue is to be placed on a frustum of a pyramid. The frustum is the part remaining after the top portion has been removed by making a cut parallel to the base of the pyramid.
- Determine the surface area of the frustum.
 - Calculate the cost of painting the frustum with gilt paint that costs $\$49.50/\text{m}^2$. It is not necessary to paint the bottom of the frustum.
17. A formula for the surface area of a rectangular prism is $SA = 2(lw + wh + lh)$.
- Suppose each of the dimensions is doubled. Show algebraically how the surface area is affected.
 - How is the volume affected if each of the dimensions is doubled? Justify your answer algebraically.
18. **Math Contest** A large wooden cube is made by glueing together 216 small cubes. Cuts are made right through the large cube along the diagonals of three perpendicular faces.
- How many of the small cubes remain uncut?

